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Design research on mathematics education: Assisting elementary school children in grasping multiplication concept and solving three-dimensional multiplication principle problems

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Abstract. The present research which is the continuance of the two-dimensional problem activities, addresses how the designed instructional learning support students in solving three-dimensional multiplication problems as well as grasping the concept of multiplication. Design research, which fits to answer the research question was chosen to reach those set objectives. Specifically, a hypothetical learning trajectory (HLT) was set consisting of learning goals, student activities, and the conjecture of their thinking and learning by applying the principle of Realistic Mathematics Education. Reflecting from the activities, the findings suggest that by creating hands-on activity, the students are able to make all different possible combinations of each kind of object. The use of tree-diagram model in the two-dimensional problems suggests the children use such kind of model.

1. Introduction

One of the topics in discrete mathematics which gets major representation in school curriculum is combinatorics [1]. Such kind of development implied educational studies in that topic also quite evolve. Some of them related to the strategies and the abilities of students used in solving combinatorial problem ([2]; [3]; [4]; [1]) , and the importance of combinatorics in solving probability problems, e.g. [5]. Some of these studies were conducted in elementary school level which motivated the researchers to introduce combinatorics in such education unit since, based on the curricula, combinotorics is firstly studied in high school level.

The topic in the combinatorics which is perceived appropriate to the elementary school children is multiplication principle which states that if there are m items in each of n and there are n items, then there will be m multiplied by n items in total. To support the students in solving multiplication principle problems as well as reaching the multiplication concept lying in the topic, literature review was done to identify steps which can be undertaken. In the studies of students' strategies of [3] covering multiplication principle problem, there were some strategies used by elementary school children in solving both two-dimensional problems and three-dimensional problems. In addition, the efficiency of those strategies is also emphasized. The trial and error approach and the odometer pattern were respectively considered as the most inefficient and efficient strategy for two-dimensional

problem. The latter strategy which was named since it resembles the odometer in a vehicle is conceptually and closely related to the multiplication concept since if there are m items in each n and there are n items, then there will be m multiplied by n items in total. Meanwhile, in the three-dimensional strategy, the most useful strategy to the concept formation is major-minor. It is so labelled since there is a major item which is less frequently changed and paired to each minor item. Although both efficient strategies are specified, there are some representations in applying them, e.g. listing method and tree-diagram model which would be elucidated in the part of research method. The research question that is posed is that how can the designed instructional learning support students in solving three-dimensional multiplication problems as well as grasping the concept of multiplication?

2. Research Method

Although, this study simply concerns three-dimensional problems, it is the continuance of the activities of the children solving two-dimensional problem through the sequence of activities aimed to support students' understanding and skills. Design research was chosen as the method of the research. [6] stated there are three phases of design research: the preparation for the experiment, the classroom experiment, and the retrospective analyses. In the preparation phase, a hypothetical learning trajectory (HLT) was designed which comprehends of learning goals, teaching and learning activities, and conjecture of student's thinking [7]. The HLT successfully made the children apply the odometer strategy with tree-diagram model in representing the whole pairs of snack-drink combinations.

Twelve 10-12 year old students divided into four groups participated in the experiment of which each group consisted of three students. They were studying in SD Athirah, an elementary school lying in downtown area of Makassar, namely one of highly populated area in Indonesia. The number of the students in this cycle was determined so to obtain more comprehensive data. They were taken as samples by random purposive sampling technique who are heterogeneous in the term of mathematics ability. High, middle, and less mathematics ability could be found in every group. In addition, each group was set unisex since, based on the discussion with their home-room teacher, it would simplified the students work cooperatively to their group mates if the students worked with the students with the same sex and assert the interactivity characteristic of RME. Furthermore, the students haven't studied multiplication principle although they have studied some basic mathematics operations, e.g. addition and multiplication. The researches acted as the HLT designer and also the teacher in the experiment phase.

The data in this study were obtained from the preparation of the experiment and the experiment. They were gathered by doing an interview, observing, and collecting written documentation. The interview and the observations were recorded by using field note and video-camera to collect some information such as the year of children, how they learned multiplication in the previous level, and the documents which were mainly collected in the experiment phases, i.e. student's written works.

Related to the previous section concerning the efficient strategies representation tree-diagram model is more considerably favorable since, besides that it looks systematic, several mathematics discrete textbooks in which the multiplication principle is covered and pedagogic literatures in teaching multiplication e.g. [8] suggested the researchers to introduce a multiplication model i.e. tree diagram model to evoke students to come up with the multiplication concept. The HLT as shown in the table 1 was set based on the theory of [9] which put forward though that the number of objects is should be firstly simplified. Then, in the HLT, it starts from 1-2-3 addressing the number of kind of snack-the number of kinds of drinks-the number of kinds of fruits. Moreover, such kind of the composition is deemed supporting for children to apply major-minor strategy. The HLT was also completed with the guide of teacher in giving feed-back toward what have been done by the children.

The use of model in this study was also inspired by Realistic Mathematics Education (RME) as the approach in designing learning of this study is underlined by its functions which not only offers a pedagogical and didactical philosophy on teaching and learning mathematics but also designing instructional materials for learning [7]. [10] suggested that the model itself is a representation made from the situation of the problem given in which there is mathematical concept. The other RME

characteristic adapted to the study is the use of familiar context delivered to the outset of the learning and the hands-on activity of which students use physical object incorporated to the learning [11].

Table 1. The HLT for Learning Experiment

Activity	Goal	Problem	Conjecture of students' thinking and learning
1	Students can list and determine the number of all possible three-dimensional pair combinations using major-minor strategy	1 snack - 2 different drinks – 3 different fruits	Some students will use *major-minor* strategy with tree diagram model Some students will use *major-minor* strategy with listing method The other students will use trial and error strategy In determining the number of the combinations, most students count the combination one by one. *Teacher guide*: Teacher let each group present its answer in the whiteboard and ask them to compare which answer simpler and more effective. In this case, teacher explains more the answer and uses the tree diagram model aiming to bridge the problem to the multiplication concept. Teacher firstly "group" and multiplies the major and the minor component as for the single major component there are some minor components and then for each group, there are some other components
		2 different snacks - 2 different drinks – 3 different fruits	Most students will use *major-minor* strategy with the tree diagram model. They simply continue the work from the previous problem. The other students will use *trial and error* strategy. They count the combinations one by one to get the total. In determining the number of the combinations, some students just make an addition or counting one by one. Meanwhile, less students grasp the concept and apply multiplication, i.e. $2 \times 2 \times 3$

The issues of the validity and the reliability in this study mainly refer to the study of [12] and the study of [7] of which internal validity, external validity, internal reliability, and external reliability should be noticed. They are all concerned in qualitative way. Internal validity refers to the data collection quality and the considerable reasoning which can be used to draw conclusion. Then in this study, it was gained by collecting the different types of data (data triangulation) such as video recording, audio recording, photographs, field notes, and written work of the students. We also

conducted different teaching experiments in the first cycle and in the second cycle aimed, one of them, to test the conjectures set in the earlier experiment in the later experiment. External validity or the generalizability is the extent to which one can generalize the findings from the contexts used in this study to other contexts which can be issued by presenting the findings of this study clearly so others can transfer it to their domains. Internal reliability means the extent to which the inference and the argumentation are reasonable. In this study, it was improved by discussing crucial activities with colleagues to minimize the sense of subjectivity and doing careful collection to the data e.g. coding the audio transcript and making video fragment. External reliability means *replicability* which has a criterion i.e. *trackability* of which a researcher should report the succession of his research in such a way that a reader can track his activities during research.

3. Results

There were sort of different process in obtaining the list of the combination answer, i.e. major-minor-listing method, tree-diagram model, and trial and error. The teacher then let the group which used tree-diagram model and that which used major-minor-listing method to present their answers in the whiteboard aiming to use it to lead the students using multiplication. The group which used tree diagram model made the snack, i.e. beng-beng as the major part and the fruits as the minor part. That group which used major-minor-listing method also made beng-beng as the major part, however, and the drinks as the minor part. No missing combination found in all the groups' work and determined the number of the combination by counting.

Figure 1. The work of students with tree-diagram model

The teacher then made a tree-diagram model, in contrast to the work of the group, and set the drinks as the minor part. The teacher asked the students whether the number of the was also six and then the students considered that it was also six by counting. The teacher then used the second problem in the HLT, 2-2-3 problem. Most students used tree-diagram model and counted ony by one the combination to obtain the total. Till the last problem, there was no indication that the students grasped the concept of multiplication in the three-dimensional problems.

Most of the literatures evoking students to take the advantage of model and apply mathematics concept aiming not to use an exhaustive process like counting, tend to make the problems more complicated, e.g. increasing the number of objects teacher ([13], [14]). The teacher then decided to add the problem of which the number of drinks alters becoming three. Before the students worked with the model, the teacher initiated to ask the groups whether they know already the number of the

combinations. Most of them already realized the number although they had not created the tree-diagram model for the context. They could imagine from the tree-diagram model they set from the previous problem, i.e. 2-2-3 problem. The teacher observed one of the groups and asked them resulting to discussion as recorded and transcribed in the following fragment:

Teacher : how many combinations in total?
Students : eighteen
Teacher : how do you know that it is eighteen?
Students : (pointing the already made tree-diagram model, exactly the minor part of the previous problem) it will be three. So this is three, three, three, three and (pointing the latest part of the tree diagram model since there was an addition one minor part, i.e. from two to three) this is three, three, three, and three
Teacher : so, what is the process in obtaining eighteen?
Students : (counting) one, two, three, then being added and so on until eighteen

Counting one by one method was also made by the other groups. Later on, in making the last attempt, since the limited time alloted for learning, to lead the students come up with the multiplication concept, the teacher made a whole discussion. Specifically, the teacher put emphasis on the number of minor part each major part has and the number of the last part each major-minor part have and the relationship among the problems related to multiplication. Starting from reexplaining the answer of 1-2-3 problem, i.e. the answer is six, the teacher then asked the students that how many combination if the number of snack becoming two. The students answered twelve since the new snack corresponded also to the six combination of drinks-fruits.

The researchers assumed that, if the problem was developed by altering the number of the snack becoming three, then the students would simply did binary operation, i.e. three times six which was considered that it would not lead the students to the concept of tertiary multiplication as there were three numbers in three dimensional problem. Then, the teacher decided to increase the number of drinks becoming three, i.e. 2-3-3. Most students then skillfully answered by using tree-diagram model of which the snack and the drink were the major and the minor component and, however, counted the combination one by one to get eighteen. Next, the teacher asked them to use another method in determining the number of the combination. The student, who answered using odometer strategy from the beginning and proficiently used multiplication for the two dimensional problems answered using multiplication for the latest problem. His explanation to the teacher and the other students was recorded in the following fragment:

Student : it is eighteen
Teacher : why is it eighteen?
Student : since it is six (pointing out the number of snacks and drinks)
Teacher : how do you get six?
Student : two times three
Teacher : why is it two multiplied by three?
Student : because one snack is paired to three drinks and there are two snacks, there are six combinations
Teacher : then go on
Student : these six pairs are paired to three fruits, so six multiplied by three equals eighteen

The student who explained the present answer didn't take the advantage the tree-diagram model available in the whiteboard by the previous student, instead, relying on the number of objects written by the teacher in the whiteboard as the figure 2 suggests.

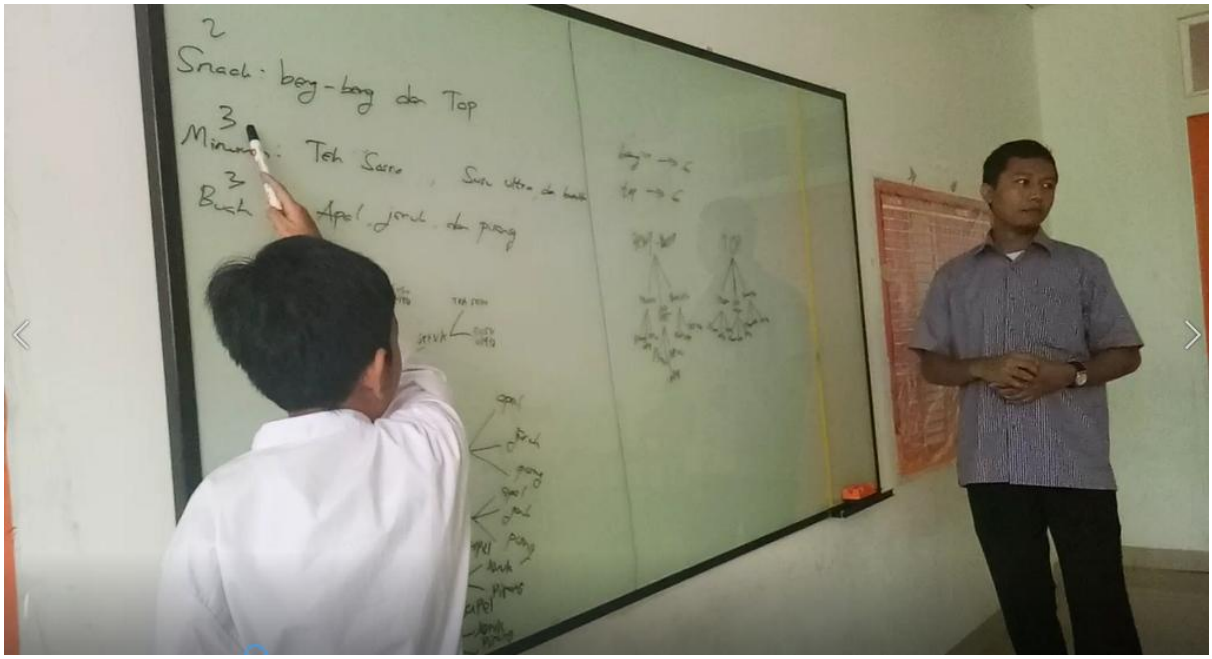

Figure 2. The student takes the advantage of the numbers of the objects to use multiplication

4. Conclusion

This paper was initially challenged with the question that how can the designed learning activities support elementary school children to apply efficient strategies in solving problem as well as to reach the understanding of multiplication principle concept?. Several conclusions were quite similar to those of the second cycle of the present design research. The hands-on activities in the beginning assist the students cover the whole combination of objects as well as lead them to apply major-minor strategy. The introduction of major-minor strategy in the form of tree-diagram by the teacher influences the students choice of representing the combination of objects. They all eventually prefer the tree-diagram model because of its simplicity for large number of objects. Furthermore, some students who skillfully used the tree-diagram model from the beginning keen on using the model in solving three dimensional problems. However, the model doesn't help the students grasp multiplication concept unless they are guided by the explanation of the teacher about how many objects the major-minor component has.

It should be noted that, although the students, based on an interview said that they mainly learned multiplication by memorizing, most of them see that multiplication as repeated addition. That understanding plays important role of the concept grasping in the learning. Although the goals of the learning are reached, the teacher focuses only on the students who follow the learning trajectory and tend to reach the learning goals smoothly and, based on the information from the school official, have high mathematics ability. The HLT simply tends to influence the other students by showing them the comparison of their answers and the sophisticated answers by their friends. It is suggested for further research to highlight the students having lack of mathematics abilities to guide them in grasping the desired mathematics concept.

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